Foundation Mathematics 1017SCG
Week 2 Summary Sheet

Logarithms

$$N = a^x \Leftrightarrow \log_a(N) = x$$

$$(\text{where } N > 0, a > 0, a \neq 1)$$

- $\log_a(M \times N) = \log_a(M) + \log_a(N)$
- $\log_a\left(\dfrac{M}{N}\right) = \log_a(M) - \log_a(N)$
- $\log_a\left(M^p\right) = p \times \log_a(M)$
- $\log_a(a) = 1$
- $\log_a(1) = 0$

$\log_e(x)$ can also be written as $\ln(x)$ and is known as the natural logarithm

Examples

$$\log_6(36) = 2$$
$$(\text{as } 6^2 = 36)$$

$$\log_2(8) = 3$$
$$(\text{as } 2^3 = 8)$$

Examples Simplify the following

$$\begin{aligned}\log_{10}(2) + \log_{10}(50) &= \log_{10}(2 \times 50)\\ &= \log_{10}(100)\\ &= 2\end{aligned}$$

$$\begin{aligned}\log_5(75) - \log_5(3) &= \log_5\left(\frac{75}{3}\right)\\ &= \log_5(25)\\ &= 2\end{aligned}$$

$$\begin{aligned}\frac{\log_{10}(x^3)}{\log_{10}(x)} &= \frac{3\log_{10}(x)}{\log_{10}(x)}\\ &= 3\end{aligned}$$

Change of Base Rule

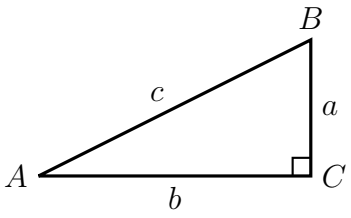
The change of base rule is helpful for converting logarithms to base 10 (or base e) so that they can be evaluated by a scientific calculator.

$$\log_b(N) = \frac{\log_a(N)}{\log_a(b)}$$

Example Calculate $\log_6(40)$ to two decimal places.

$$\begin{aligned}\log_6(40) &= \frac{\log_{10}(40)}{\log_{10}(6)}\\ &= 2.06\end{aligned}$$

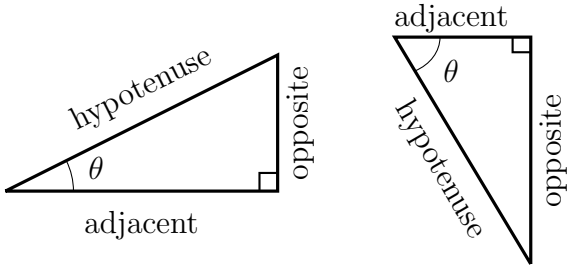
Pythagoras' Theorem



$$a^2 + b^2 = c^2$$

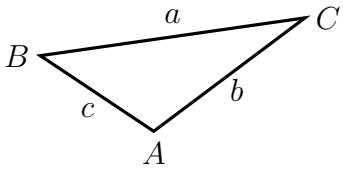
Note that when using this formula, c must be the hypotenuse of the triangle (the longest side of the right-angled triangle).

Trigonometry



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Sine and Cosine Rules



The sine and cosine rules can be used when working with non-right-angled triangles.

Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$