Foundation Mathematics 1017SCG
Week 5 Summary Sheet

Solving Linear Equations

Linear equations (where the variable is only to the power of one) can be solved by rearranging. Remember, whatever is done to one side of the equation **must also** be done to the other side of the equation.

Solve $6x - 5 = 4x + 1$

$$\begin{aligned} 6x - 4x - 5 &= 4x - 4x + 1 && \text{subtract } 4x \\ 2x - 5 &= 1 && \text{simplify} \\ 2x - 5 + 5 &= 1 + 5 && \text{add } 5 \\ 2x &= 6 && \text{simplify} \\ \frac{2x}{2} &= \frac{6}{2} && \text{divide by } 2 \\ x &= 3 \end{aligned}$$

Solving Quadratic Equations

Quadratic equations of the form $ax^2+bx+c = 0$ can be solved using factoring or the quadratic formula. A quadratic equation can either have

- Two unique real solutions
- One unique real solution
- No real solutions (complex solutions only)

Examples using factoring

Solve $x^2 - 5x + 6 = 0$

Start by factoring the quadratic.

$$(x - 2)(x - 3) = 0$$

$$\begin{aligned} x - 2 &= 0 & x - 3 &= 0 \\ x &= 2 & x &= 3 \end{aligned}$$

Therefore the solutions are $x = 2$ and $x = 3$. (Two unique real solutions).

Example Solve $x^2 - 4x - 12 = 0$

$$(x + 2)(x - 6) = 0$$

$$\begin{aligned} x + 2 &= 0 & x - 6 &= 0 \\ x &= -2 & x &= 6 \end{aligned}$$

Therefore the solutions are $x = -2$ and $x = 6$. (Two unique real solutions).

Quadratic Formula

The quadratic formula can be used to solve a quadratic in the form $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example Solve $x^2 - 5x - 6 = 0$

$$a = 1, \quad b = -5, \quad c = -6$$

$$\begin{aligned} x &= \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times -6}}{2 \times 1} \\ &= \frac{5 \pm \sqrt{25 + 24}}{2} \\ &= \frac{5 \pm \sqrt{49}}{2} \\ &= \frac{5 \pm 7}{2} \end{aligned}$$

Therefore, $x = 6$, $x = -1$.

Solving Logarithmic Equations

$$N = a^x \;\Leftrightarrow\; \log_a(N) = x$$

$$(\text{where } N > 0,\ a > 0,\ a \neq 1)$$

$$a^{\log_a(N)} = N$$

Solve $\log_5(3x) = 2$

$$\begin{aligned} 3x &= 5^2 && \text{definition of logarithm} \\ 3x &= 25 && \text{simplify} \\ \frac{3x}{3} &= \frac{25}{3} && \text{divide both sides by } 3 \\ x &= \frac{25}{3} \end{aligned}$$

Solving Exponential Equations

Exponential equations (indicial equations) are solved using logarithms.

Solve $4^x = 20$

$$\begin{aligned} \log_{10}(4^x) &= \log_{10}(20) && \text{log of both sides} \\ x\log_{10}(4) &= \log_{10}(20) && \text{using log laws} \\ \frac{x\log_{10}(4)}{\log_{10}(4)} &= \frac{\log_{10}(20)}{\log_{10}(4)} && \text{divide both sides} \\ x &\approx 2.16 && \text{using calculator} \end{aligned}$$

The above equation was solved using logarithms with base 10. However, logarithms with other bases could have been used to solve the equation (for example, base e or base 4).