Foundation Mathematics 1017SCG
Week 6 Summary Sheet

Exponential Growth and Decay

Exponential growth and exponential decay can be modelled using the following equation.

$$N = N_0 e^{kt}$$

- t is the amount of time passed
- N is the amount present at time t
- N_0 is the initial amount (at $t = 0$)
- k is the growth/decay factor
- $e \approx 2.718$ (e is irrational)

The growth/decay factor k can either be positive, negative or zero.

- $k > 0$, N will increase over time (growth)
- $k < 0$, N will decrease over time (decay)
- $k = 0$, N will remain constant

Example The number of fish in a small pond can be modelled using an exponential growth model. Initially, there were 50 fish in the pond. After 15 weeks, the number of fish in the pond had increased to 70. Using the exponential growth model, how many fish are expected to be in the pond after 25 weeks?

Substituting the known information into the exponential model gives

$$70 = 50e^{k\times 15}$$

We can now solve for the growth factor k.

$$\begin{aligned} \frac{70}{50} &= e^{15k} \\ \ln\left(\frac{70}{50}\right) &= \ln\left(e^{15k}\right) \\ \ln\left(\frac{70}{50}\right) &= 15k\ln(e) \\ k &= \frac{1}{15}\ln\left(\frac{70}{50}\right) \\ k &\approx 0.0224 \end{aligned}$$

Now that the growth factor k has been calculated, the model can be used to predict the number of fish in the pond after 25 weeks.

$$\begin{aligned} N &= 50 \times e^{0.0224\times 25} \\ &\approx 87.53 \end{aligned}$$

Therefore, after 25 weeks, there would be approximately 88 fish in the pond.

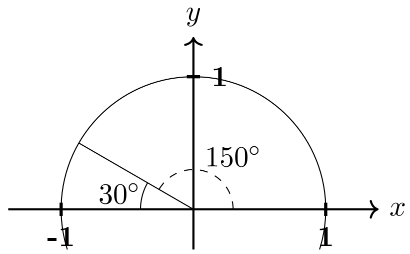
Solving Trigonometric Equations

Trigonometric equations usually have **many** solutions.

Example Solve $\sin(\theta) = \frac{1}{2}$ where $0° \leq \theta \leq 360°$

Start by finding the solution in the first quadrant. Using the special triangles, $\theta = 30°$.

There is also a solution in the second quadrant (as sin is positive in the first and second quadrants). There will be no solutions in the third or fourth quadrants.

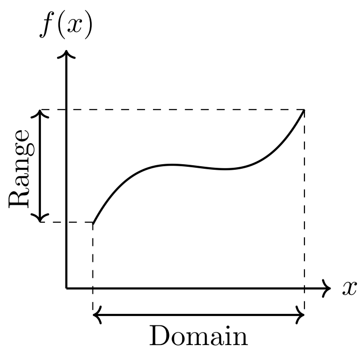


Therefore, $\theta = 30°$ and $\theta = 150°$.

Domain and Range

Consider the function $f(x)$

- The **domain** of $f(x)$ is the set of all possible values of x for which the function is defined (i.e. all the values of x that can be used with the function)
- The **range** of $f(x)$ is the set of all possible values that can be returned by the function



When determining the domain of a function, consider if any input values will cause a 'problem' where the function is unable to give a sensible output.

- Will an input cause the square root of a negative number?
- Will an input cause the logarithm of a negative number?
- Will an input cause a divide by zero?