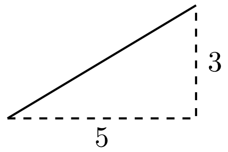
Gradient (Slope)

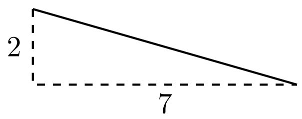
$$\text{gradient} = m = \frac{\text{rise}}{\text{run}}$$

The rise is 3 (y direction) and the run is 5 (x direction). Therefore the gradient of the line is $\frac{3}{5}$.



$$m = \frac{3}{5}$$

The rise is -2 (moved **down** 2 in the y direction) and the run is 5 (x direction). Therefore the gradient of the line is $-\frac{2}{5}$.



$$m = -\frac{2}{7}$$

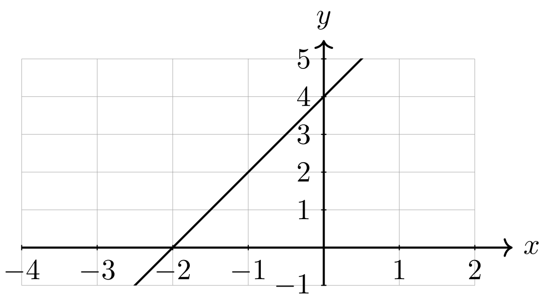
Graphing Linear Equations

A linear equation forms a straight line when graphed on the cartesian plane. Consider the linear equation $y = mx + c$ (gradient-intercept form).

- m is the gradient (slope) of the line
- c is the y-intercept

The x-intercept can be found by letting $y = 0$. Once the x-intercept and y-intercept have been found, they can be plotted on the cartesian plane and a line drawn through both intercepts.
Example Draw a graph of $y = 2x + 4$.
As the equation is already in gradient-intercept form, the gradient is 2 and the y-intercept is 4.
To find x-intercept, let $y = 0$.

$$\begin{aligned} 0 &= 2x + 4 \\ -2x &= 4 \\ x &= -2 \end{aligned}$$



Finding the Equation of a Line

The equation of a straight line can be found using

$$y - y_1 = m(x - x_1)$$

where m is the gradient of the line and (x_1, y_1) is a point that the line passes through.

Finding the Gradient of a Line

The gradient of a linear line can be found using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m is the gradient of the line and (x_1, y_1) and (x_2, y_2) are points that the line passes through.

Parallel and Perpendicular Lines

Parallel lines have the **same gradient**. Perpendicular lines have gradients that **multiply to give -1**.

- $y = 3x - 5$ and $y = 3x + 7$ are parallel
- $y = -4x + 1$ and $y = \frac{1}{4}x - 6$ are perpendicular

Simultaneous Equations

Example Solve the following simultaneous equations.

$$\begin{aligned} 2x + y &= 4 && (1) \\ 5x + 4y &= 13 && (2) \end{aligned}$$

Step 1 Rearranging (1) gives $y = 4 - 2x$.
Step 2 Substituting $y = 4 - 2x$ into (2) gives

$$\begin{aligned} 5x + 4(4 - 2x) &= 13 \\ 5x + 16 - 8x &= 13 \\ -3x &= -3 \\ x &= 1 \end{aligned}$$

Step 3 Substituting $x = 1$ into $y = 4 - 2x$ gives $y = 4 - 2 \times 1 = 2$. Therefore the solution to the simultaneous equations is $x = 1, y = 2$.

Graphing Quadratic Equations (Parabola)

Consider the quadratic equation $y = ax^2 + bx + c$.

- The **turning point** is given by $x = -\frac{b}{2a}$
- The y-intercept is c
- The x-intercepts can be found by letting $y = 0$ and then solving the quadratic equation using factoring or the quadratic formula

The Discriminant

The discriminant, D, can be used to determine the number of x-intercepts of the quadratic equation $y = ax^2 + bx + c$.

$$D = b^2 - 4ac$$

- $D > 0$: There are two unique x-intercepts
- $D = 0$: There is one x-intercept
- $D < 0$: There are no real x-intercepts