Foundation Mathematics 1017SCG
Week 8 Summary Sheet

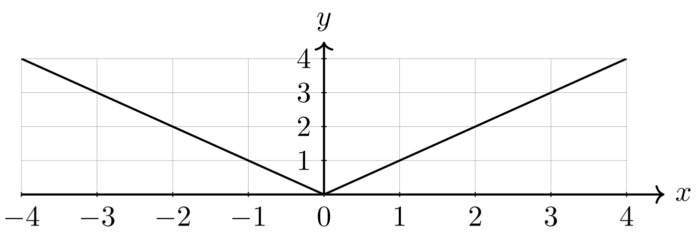
Absolute Value

The absolute value of x, written as $|x|$, is the distance of x from zero, where direction is not important.

$$|7| = 7, \qquad |-5| = 5, \qquad |0| = 0$$

Graphing Absolute Value Functions

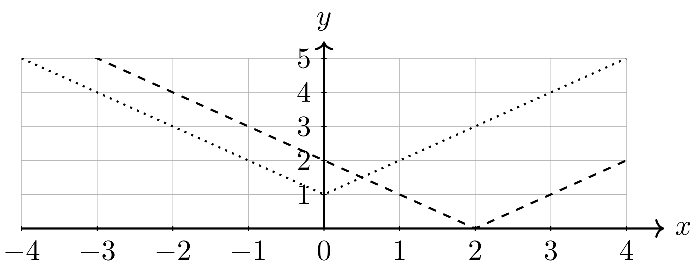
A graph of the function $y = |x|$ is shown below.



Consider the graph of $y = |x + F| + G$

- F is the **horizonal shift**.
  If $F > 0$, the graph has been shifted to the left.
  If $F < 0$, the graph has been shifted to the right.
- G is the **vertical shift**.
  If $G > 0$, the graph has been shifted up.
  If $G < 0$, the graph has been shifted down.

Graphs of the functions $y = |x - 2|$ (dashed line) and $y = |x| + 1$ (dotted line) are shown below.



The graph of $y = -|x|$ is a **reflection** of the graph $y = |x|$ across the x-axis.

Test for Functions

The **vertical line test** can be used to determine if a curve is the graph of a function. If any vertical line cuts the curve at most once, then it is a function. If a vertical line cuts the graph more than once, it is not a function.

Piecewise (Hybrid) Functions

Consider the following piecewise function.

$$f(x) = \begin{cases} x^2 + 4 & x < 2 \\ 5x - 7 & x \geq 2 \end{cases}$$

When evaluating a piecewise function, we must first determine which 'piece' of the function we are using based upon the value of x.
Example $f(1) = 1^2 + 4 = 5$
Example $f(3) = 5 \times 3 - 7 = 8$

Inverse Functions

Consider the function $f(x)$. Its inverse function, $f^{-1}(x)$, 'undoes' or 'reverses' the effect of $f(x)$.

$$f^{-1}(f(x)) = x$$

When graphed, the function $f(x)$ and its inverse $f^{-1}(x)$ are reflections across the line $y = x$.

Test for Inverse Functions

Not all functions have inverses. The **horizontal line test** can be used to determine if a function has an inverse. Consider the graph of the function $f(x)$. If any horizontal line cuts the graph at most once, the function $f(x)$ has an inverse. If a horizontal line cuts the graph more than once, the function $f(x)$ does not have an inverse.

Finding the Inverse of a Function

Given the function $f(x)$ (where $f(x)$ has an inverse), its inverse function can be found by rearranging for x.
Example Given $f(x) = \frac{x-2}{5}$, find $f^{-1}(x)$.
First, let $y = f(x)$ (y is easier to work with than $f(x)$)

$$y = \frac{x-2}{5}$$

Rearrange to find an expression for x in terms of y.

$$5y = x - 2$$
$$x = 5y + 2$$
$$f^{-1}(y) = 5y + 2$$

We have now found the inverse function, but it is terms of y. The final step is to write the inverse function in terms of x.

$$f^{-1}(x) = 5x + 2$$

Composite Functions

Consider the two functions $f(x)$ and $g(x)$. The composite function $f(g(x))$ is equivalent to starting with x, applying the function g to give $g(x)$ and then applying the function f to give $f(g(x))$.
Example Given $f(x) = x^2$ and $g(x) = x + 5$, find the composite functions $f(g(x))$ and $g(f(x))$.

$$\begin{aligned} f(g(x)) &= f(x + 5) \\ &= (x + 5)^2 \\ g(f(x)) &= g(x^2) \\ &= x^2 + 5 \end{aligned}$$