Foundation Mathematics 1017SCG
Week 10 Summary Sheet

Definition of Derivative

Consider the function $y = f(x)$. The **derivative** or **instantaneous rate of change** of the function is written as $\frac{dy}{dx}$ and is given by

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of $f(x)$ can also be written as $f'(x)$.

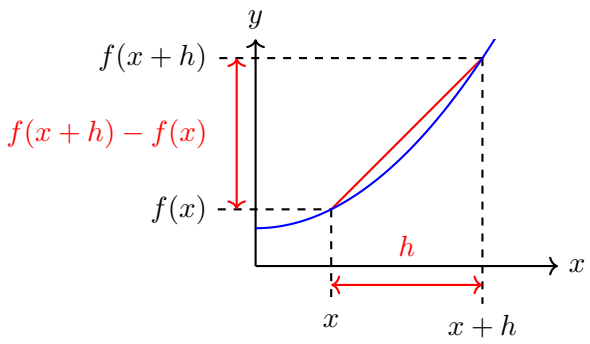

Table of Derivatives

In the table below, n and c are constants.

y	$\frac{dy}{dx}$
c	0
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$

Finding Turning Points Using Derivatives

At a turning point (minimum or maximum), the derivative is zero. $\left(\frac{dy}{dx} = 0\right)$

Example Find the turning point of $y = x^2 - 8x + 6$. We will start by finding the derivative.

$$\frac{dy}{dx} = 2x - 8$$

To find turning point, let $\frac{dy}{dx} = 0$.

$$2x - 8 = 0$$
$$2x = 8$$
$$x = 4$$

The y value of the turning point can be found by substituting $x = 4$ into the original function.

$$y = 4^2 - 8 \times 4 + 6 = -10$$

Therefore, the turning point is at $x = 4$, $y = -10$.

Tangent Line

Consider finding the tangent to the function $y = f(x)$ at the point $x = a$.

- The tangent line is a straight line which can be found using $y - y_1 = m(x - x_1)$.
- The tangent line passes through the point $f(a)$. *It passes through the same point as the original function.*
- The tangent line has gradient of $f'(a)$. *It has the same gradient as the original function.*

Example Find the tangent to $y = 2x^3$ at $x = 1$.

$$\text{When } x = 1, y = 2 \times 1^3 = 2$$

Therefore, the tangent passes through the point (1,2). To find gradient, we need to find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 6x^2$$
$$\text{When } x = 1, \frac{dy}{dx} = 6 \times 1^2 = 6$$

Therefore, the tangent has a gradient of 6.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 6(x - 1)$$
$$y - 2 = 6x - 6$$
$$y = 6x - 4$$

Therefore, the tangent line is $y = 6x - 4$.

Greatest and Least Value

Consider the continuous function $y = f(x)$ on the finite interval $a \leq x \leq b$. The **greatest and least** values of the function will occur at one of the following:

- a turning point
- the end points of the domain ($x = a$ or $x = b$)
- where the derivative is not defined (advanced)