Foundation Mathematics 1017SCG
Week 11 Summary Sheet

Table of Integrals

In the table below, k, n and C are constants.
(C is the constant of integration).

$f(x)$	$\int f(x)\ dx$
k	$kx + C$
x^n	$\frac{x^{n+1}}{n+1} + C$
$\sin(x)$	$-\cos(x) + C$
$\cos(x)$	$\sin(x) + C$
e^x	$e^x + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$

Indefinite Integrals

Example

$$\int 5x^2 + 4\sin(x) + 7e^x\ dx = \frac{5x^3}{3} - 4\cos(x) + 7e^x + C$$

Definite Integrals

Given that $\int f(x)\ dx = F(x)$, a definite integral can be calculated using

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

where a is the lower limit of integration and b is the upper limit of integration.

Example

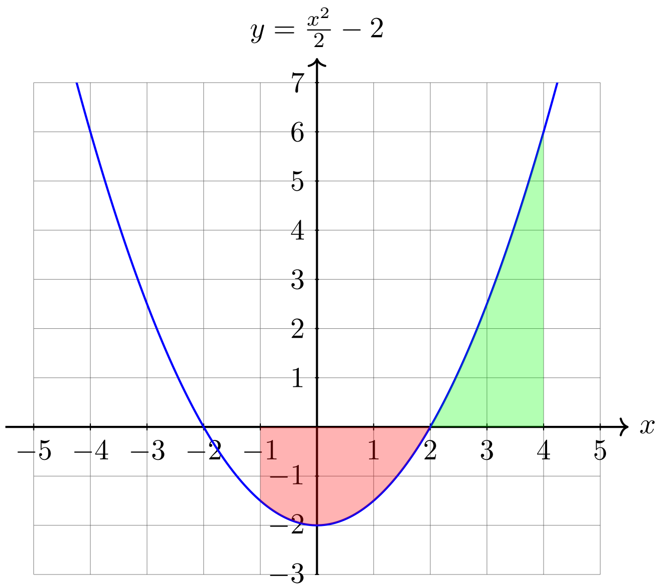
$$\begin{aligned}\int_1^4 6x^2\ dx &= \left[\frac{6x^3}{3}\right]_1^4 \\ &= \left[2x^3\right]_1^4 \\ &= \left(2\times 4^3\right) - \left(2\times 1^3\right) \\ &= 128 - 2 \\ &= 126\end{aligned}$$

Note that it is not necessary to include the constant of integration when evaluating definite integrals.

Area under Curve

A definite integral can be used to calculate the area bounded by the curve and x-axis.

- Area above the x-axis is considered positive (green shaded region)
- Area below the x-axis is considered negative (red shaded region)



Example Find the area bounded by the curve $y = \frac{x^2}{2} - 2$ and the x-axis between $x = 2$ and $x = 4$.

$$\begin{aligned}\int_2^4 \frac{x^2}{2} - 2\ dx &= \left[\frac{x^3}{6} - 2x\right]_2^4 \\ &= \left(\frac{4^3}{6} - 2\times 4\right) - \left(\frac{2^3}{6} - 2\times 2\right) \\ &= \left(\frac{8}{3}\right) - \left(-\frac{8}{3}\right) \\ &= \frac{16}{3}\end{aligned}$$

The area of the green shaded region is $\frac{16}{3}$ units^2.

Example Find the area bounded by the curve $y = \frac{x^2}{2} - 2$ and the x-axis between $x = -1$ and $x = 2$.

$$\begin{aligned}\int_{-1}^2 \frac{x^2}{2} - 2\ dx &= \left[\frac{x^3}{6} - 2x\right]_{-1}^2 \\ &= \left(\frac{2^3}{6} - 2\times 2\right) - \left(\frac{(-1)^3}{6} - 2\times(-1)\right) \\ &= \left(-\frac{8}{3}\right) - \left(\frac{11}{6}\right) \\ &= -\frac{9}{2}\end{aligned}$$

The area of the red shaded region is $\frac{9}{2}$ units^2.